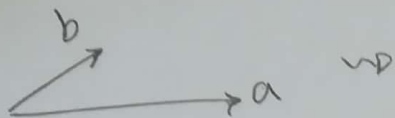
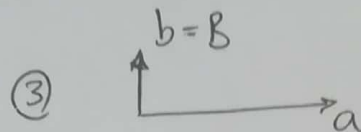
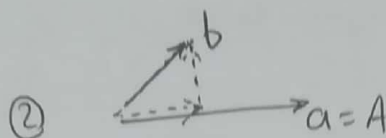


QR Decomposition



① $\rightarrow a = A$



$a, b, c \rightarrow A = a$

$$B = b - \frac{A^T b}{A^T A} A$$

$$C = c - \frac{A^T c}{A^T A} A - \frac{B^T c}{B^T B} B$$

$$q_1 = \frac{A}{\|A\|}, \quad q_2 = \frac{B}{\|B\|}, \quad q_3 = \frac{C}{\|C\|}$$

$$Q = [q_1 \ q_2 \ q_3]$$

$$[a \ b \ c] = A = QR = [q_1 \ q_2 \ q_3] R \quad \text{upper triangular}$$

SVD Decomposition

$$A \in \mathbb{R}^{m \times n}$$

$$\begin{cases} A A^T = U \Lambda U^T \\ A^T A = V \Lambda V^T \end{cases}$$

symmetric

$$A = U \Sigma V^T$$

$$V^T u_i = A v_i \quad \begin{matrix} \text{row space} \\ \text{col space} \end{matrix}$$

$$\Lambda = \Sigma^2$$

* Random Variables:

Function: $\underbrace{\{\text{outcome}\}}_{\text{Val}(X)} \rightarrow \mathbb{R}$ (e.g. dice) $\rightarrow P(X=1)=P(X=2)=\dots=P(X=6)=\frac{1}{6}$

* Conditional Distribution:

$$P(X=a | Y=b) = \frac{P(X=a, Y=b)}{P(Y=b)}$$

e.g.

$\sqrt{\square}$ dice is odd (Y)
 \square $X=1$ $\Rightarrow P(X=1 | Y) = \frac{P(X=1, Y)}{P(Y)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$

* Independence

$$P(X, Y) = P(X)P(Y) \quad / \quad X \perp Y$$

$$P(X, Y | Z) = P(X | Z)P(Y | Z) \quad / \quad P(X | Z) = P(X | Y, Z)$$

* Bayes Rule

$$P(X | Y) = \frac{P(Y | X)P(X)}{P(Y)}$$

* Expectation

$$E_P[X] = \sum_{x \in \text{Val}(X)} P(x) x, \quad \int x f(x) dx$$

* Variance

$$\text{Var}(X) = E[(X - E[X])^2] \stackrel{?}{=} E[X^2] - E[X]^2$$

* Estimation

Problem? Find some parameter θ using some data D

- We assume records in D are indep.

Maximum Likelihood Estimation (?)

$$\text{Maximize } p(D|\theta) = \text{Max} \prod_{i=1}^N p(d_i|\theta)$$

MAP Estimation (?)

$$\text{Max } p(\theta|D) = \text{Max} \frac{\overset{\text{likelihood}}{p(D|\theta)} \overset{\text{prior}}{p(\theta)}}{\underset{\text{Constant w.r.t. } \theta}{p(D)}}$$

$$\propto \text{Max } p(D|\theta) p(\theta) = \text{Max} \left[\prod_{i=1}^N p(d_i|\theta) \right] p(\theta)$$

Prediction under Posterior

$$p(x|D) = \int p(x|\theta) p(\theta|D) d\theta$$

\downarrow
 event we want to predict

\downarrow
 All possible θ

\nearrow How sure we are